FM

Internal Note No. 69-FM-256

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MSC INTERNAL NOTE NO. 69-FM-256

OCTOBER 7, 1969

MINIMUM FLIGHT TIME AND FEASIBILITY OF MULTIPLE ORBIT TRANSFERS BETWEEN EARTH AND MARS AND BETWEEN EARTH AND VENUS

Advanced Mission Design Branch
MISSION PLANNING AND ANALYSIS DIVISION

MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

(NASA-TM-X-70006) MINIMUM FLIGHT TIME AND FEASIBILITY OF MULTIPLE ORBIT TRANSFERS BETWEEN EARTH AND MARS AND BETWEEN EARTH AND VENUS (NASA) 20 p

N74-72471

Unclas
00/99 16740

MSC INTERNAL NOTE NO. 69-FM-256

MINIMUM FLIGHT TIME AND FEASIBILITY OF MULTIPLE ORBIT TRANSFERS BETWEEN EARTH AND MARS AND BETWEEN EARTH AND VENUS

By Ellis W. Henry
Advanced Mission Design Branch

October 7, 1969

MISSION PLANNING AND ANALYSIS DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

MANNED SPACECRAFT CENTER

HOUSTON, TEXAS

Victor R. Bond

Approved: for
Jack Funk, Chief
Advanced Mission Design Branch

Approved:
John P. Mayer, Chief
Mission Planning and Analysis Division

CONTENTS

MINIMUM FLIGHT TIME AND FEASIBILITY OF MULTIPLE ORBIT TRANSFERS BETWEEN EARTH AND MARS AND BETWEEN EARTH AND VENUS

By Ellis W. Henry

1.0 INTRODUCTION

The minimum flight time for multiple orbit transfers from Earth to Mars (or Venus) and from Mars (or Venus) to Earth are presented to establish lower limit values for consideration with that type of interplanetary trajectory. The multiple orbit transfers, abbreviated here as M. O. trajectories, are those in which the angular displacement of the spacecraft exceeds 360° during the time of transfer between two planets. This displacement is in contrast to direct trajectories in which the displacement is less than 360°. The M. O. Trajectory must be along a heliocentric ellipse which has an apoapsis R_a that is greater than or equal to the orbital radius of the outer planet and has a periapsis R_p that is less than or equal to the orbital radius of the inner planet. That is, the ellipse must be tangent to or must cross the orbital path of each planet, and the spacecraft must follow this elliptical path for more than one revolution.

A schematic of an M. O. transfer is included; the method for evaluation of the minimum flight time for such a transfer is described; and the results of a comprehensive analysis are given by data and conclusions. The applicability and feasibility of the M. O. transfers for interplanetary missions is discussed.

2.0 DERIVATION OF THE MINIMUM FLIGHT TIME TRANSFER

An Earth to Mars M. O. trajectory is described by use of figure 1, which shows the orbits of the inner planets out to Mars and a hypothetical ellipse that intersects the orbits of Earth and Mars at the points A, B, C, and D. In the figure, angle θ represents at transfer angle which is greater than 360°, which illustrates an M. O. transfer from Earth (point A) to Mars (point B). The angle θ_e which is less than

360° represents the excess of multiples of 360° in the transfer angle θ; θ_e would represent a direct transfer from Earth or Mars if the planets were at the points A and B at the proper time. The ellipse is defined mathematically by the values of R_a and R_p; that is, the major axis $2a = R_a + R_p$, and the eccentricity $e = (R_a - R_p)/(R_a + R_p)$.

The orbital period of an ellipse varies by the 1.5 power of its major axis and, therefore, has a minimum value when both R_a and R_p have their minimum values. For the Mars M. O. transfer, the minimum value of R_a is the distance of Mars from the sun [1.523 astronomical units (AU)]. Because the minimum value of R_p is determined by a spacecraft design or by an operational constraint rather than by a physical limitation, representative values were chosen for this study. These values include the heliocentric distances of Mercury (0.387 AU), of Venus (0.723 AU), and of Earth (1.000 AU) and intermediate values (0.555 and 0.862 AU). The two largest values are not applicable for Venus missions, however, because R_p cannot exceed the orbit of the inner planet.

The minimum transfer time for an M. O. trajectory from Earth to Mars is one orbital period of the ellipse, that is, the time required to traverse a 360° angle from point A to point A in figure 1 in addition to the flight time required to traverse the angle θ_e from point A to point B. Longer flight times are required if two or more revolutions are made or if the angle θ_e is increased (e.g., for a transfer from A to C or from D to B or C). Similarly, the minimum Mars to Earth transfer is from point C and once around to point D for the M. O. case. For a given ellipse, the flight time required to go directly from point A to point B and from point C to point D is the same; therefore, the M. O. transfer time is the same in each case.

For any assumed values of R_a and R_p, the period of the transfer ellipse in years is readily computed from the formula $[(R_p + R_a)/2]^{3/2}$, when both R_a and R_p are in astronomical units. Evaluations of this formula are included in various figures. Evaluation of the additional transfer time for the minimum angle θ_e (i.e., the angle between points A and B or C and D,) is derived by computation of the time from periapsis to each of the two points followed by computation of the difference between these times. This additional transfer time was determined with a computer program which sequentially computed true anomaly angle, eccentric anomaly angle, mean anomaly angle, mean motion; and time from periapsis for each point; the difference and the period were then computed. The sum of one period and the minimum additional flight time is of primary interest because this sum represents the minimum M. O. transfer time. The values of R_a and R_p which result in the minimum M. O. flight time are also of interest.

For the special case of a transfer ellipse which is tangent to both of the planet orbits (i.e., minimum R_a and maximum R_p), the M. O. transfer time is 1.5 times the period of the transfer ellipse. Note that this special case is an example of a Hohmann transfer (a 180° transfer between two circular orbits with an ellipse of minimum energy), except that in the M. O. case an additional full revolution is made before the spacecraft rendezvous with the target planet. However, the flight time can be reduced considerabley by allowing a smaller value of R_p, and can be reduced to a lesser extent by allowing a slightly larger value of R_a, as compared to the special case which can be called an M. O. Hohmann transfer.

3.0 RESULTS OF THE FLIGHT TIME EVALUATIONS

The minimum multiple orbit transfer time has been computed for each of the several values of R_p defined earlier and for a range of values of R_a which equals and exceeds the minimum value of that parameter. The results of these evaluations are given in figures 2(a) through 2(e) for Mars missions and in figures 3(a) through 3(c) for Venus missions. The results are the same whether the transfer is from Earth to the other planet or from the other planet to Earth. In figures 2 and 3 are shown the minimum direct transfer time for the defined ellipse (the lowest curve on the graph), the period of the ellipse (the intermediate curve), and the sum of these times is the minimum transfer time for the values of R_a and R_p. Specific values of R_p are used for each of the separate figures; the smaller values of R_p result in the shortest M. O. transfer time, but a realistic spacecraft limitation causes one of these figures to be selected as the acceptable minimum. On any figure selected for consideration, as the value of R_a is increased above its minimum value, note that the period increases according to the simple formula given earlier. However, the additional flight time for the direct transfer or for the excess of one revolution decreases because of a greater velocity at points A and B. For a very small increase in

R_a above its minimum value, the additional flight time decreases more rapidly than the period increases; consequently, the sum of the two flight times decreases. However, for significantly larger values of R_a, the converse is true and the minimum M. O. transfer time increases rapidly. Each point on the upper curve represents the minimum M. O. transfer time for the corresponding value of R_a; the lowest point on the curve indicates the absolute or idealistic minimum M. O. time and the value of R_a requisite to achieve it.

The lowest curve can also be used to show the minimum value of the required transfer time for a direct trajectory with the indicated values of R_a and R_p; however, there are no limits to the values of these parameters for the direct case which can be hyperbolic or elliptical. The minimum value pertains to the minimum transfer angle case or as in figure 1, the case of a transfer from point A to point B or from point C to point D through the angle θ_e.

4.0 SUMMARY OF FLIGHT TIME RESULTS

The minimum M. O. transfer time is determined predominately by the minimum acceptable value of R_p which has been termed a spacecraft limitation; only a slight reduction is achieved by variations of the value of R_a. To achieve the minimum M. O. time, it is required that the aphelion (the value of R_a) be approximately 0.05 AU greater than the orbit of the outer planet. It is further required that the angular difference between the planets permit the transfer for the angle θ_e which is the least of four possible values as shown in figure 1. The latter condition is possible only once during a synodic cycle, which in the case of Mars is approximately 780 days and in the case of Venus is approximately 584 days. Therefore, these minimum values seldom would be seen in trajectory studies unless by design. However, because R_a is not much greater than the orbit of the outer planet for the minimum value of flight time to be achieved, a transfer from point A to point C is not much longer in duration than the minimum case, which offers another opportunity in a synodic cycle. If the minimum acceptable value of R_p happens to be near the orbit of the inner planet, the multiple orbit transfer could begin at point D; the transfer time would again be increased, but additional opportunities for a multiple orbit transfer within a synodic cycle would be possible.

In summary, when constrained by a perihelion distance of no less than the orbit of Venus [fig. 2(c)], multiple orbit trajectories to or from Mars require a minimum of 567 days for an M. O. transfer; or if the trajectories to constrained by a perihelion half way between Mercury and Venus, the requirement is reduced to 500 days [fig. 2(b)]. If the trajectory is constrained by the Venus distance, Venus missions require a minimum of 412 days [fig. 3(c)]; however, if a periapsis halfway between Mercury and Venus is used, the requirement is reduced to 324 days [fig. 3(b)]. These minimum values can be achieved only once during a synodic cycle, and the requisite aphelion distance is approximately 0.05 AU greater than the orbit of the outer planet.

Because these are minimum values, interplanetary mission studies need not consider a M. O. trajectory for any flight time shorter than the values given on the appropriate figure. Also, unless the positions of the planets are ideal for the M. O. transfer, the required flight time must increase.

Only the minimum of the four possible transfer times that are less than two periods and only the ideal geometry case have been illustrated in the figures.

5.0 APPLICATION AND FEASIBILITY OF MULTIPLE ORBIT TRANSFERS IN INTERPLANETARY MISSIONS

The minimum M. O. transfer time is more than twice the time required for a minimum energy or Hohmann transfer, which for Mars is 259 days and for Venus is 146 days. However, for some considerations, the M. O. trajectories have some particularly beneficial characteristics. It is sufficient to note these characteristics by use of any assumed ideal geometry case with an approximation of the trajectories by a somewhat crude description rather than with mathematical evaluations. For example, the bending effect on a trajectory caused by a near encounter with a planet (gravity turn) can be ignored for the present purpose. Precise evaluations should lead to approximately the same conclusions.

Relatively low energy round trip missions to Venus can be achieved with a total mission duration of 2 years by use of a transfer ellipse with a period of 1 year. The spacecraft would be launched at Earth, make two revolutions, and return at Earth 2 years later. On this journey, the spacecraft would cross the orbit of Venus four times; that is, there would be four opportunities to encounter Venus with either a direct flight or a multiple orbit trajectory, followed by a return to Earth along the same ellipse. Data presented in figure 3(c) indicate that such a mission is possible. An ellipse with a period of 1 year can have $R_a = 1.277$ AU with $R_p = 0.723$ AU, or these values can be altered if their sum is the same and if R_p is less than the orbit of Venus. For a flight to Venus, the M. O. trajectory could be used followed by a direct return to Earth or vice versa. This situation provides an ideal opportunity for either a free-return flyby of Venus or a near minimum impulse velocity maneuver for capture and escape with a short stay time to complete an orbital mission to Venus. A similar situation exists for Mars missions with a total mission time of 3 years when a transfer ellipse with a period of 1.5 years is used. The data presented in figure 2(e) indicate that this technique is possible with $R_p = 1.000$ AU and $R_a = 1.625$ AU, or again with a tradeoff in these values. It is not essential that the direct flight be along the same ellipse as the M. O. trajectory, but some of the advantages are realized only if the trajectory from Earth is the direct flight along the M. O. ellipse. This case will be considered.

The 2-year Venus mission and the 3-year Mars M. O. mission have relatively short staytimes under the conditions postulated but have nearly the same total mission duration and require nearly the same energy (velocity requirements) as a double Hohmann transfer. For the latter type of transfer, only direct trajectories are used, but a long staytime at the planet is required while awaiting the opportunity for a minimum energy return to Earth. Therefore, there is an obvious tradeoff between staytime and flight time. The tradeoff is not quite precise, however, because the double Hohmann missions are slightly shorter in total duration. Also, the tradeoff involves a penalty because any mission other than a Hohmann will require more than minimum energy.

The 3-year Mars mission with an M. O. transfer is sufficiently near a minimum energy mission (because R_a exceeds the orbit of Mars only slightly) that this can be considered to be the case temporarily. If a direct trans-Mars trajectory is used, then it must be nearly a direct Hohmann transfer if the free return is nearly an M. O. Hohmann transfer; consequently, another similarity in the 3-year Mars mission compared to the direct double Hohmann is that it has a similar date to depart Earth. With a similar total mission duration, entry into the Earth atmosphere at the end of the mission will occur at a similar date. Lack of the Hohmann characteristics causes the launch dates for a Venus mission to be slipped further apart, but the requisite planetary alinement places the launch dates in similar calendar periods.

The advantage of using an M. O. transfer apparently is not that the total mission time can be shortened nor that the transfer offers any appreciably different launch opportunity; however, precise trajectory studies based on actual planetary ephemerides could show variations from the idealistic assumptions included here. There is no economy of energy (or velocity requirement) as compared to the direct double Hohmann transfer which requires only minimum energy. However, there is no appreciable penalty for choosing an M. O. trajectory for the advantages it offers. One advantage has already been suggested, the ability to achieve a free-return trajectory to the planets, which provides an inherent abort capability prior to orbit insertion at the target planet. Because of the much longer flight time of the M. O. trajectory, it does not seem feasible to use this type of transfer to the planet for an orbital mission so that a free return will be possible in case of an abort. A more feasible plan would be to use the direct trajectory which provides an M. O. return if an abort is required; then the mission is not severely penalized if the abort is not required. The latter choice provides another option, again in terms of a favorable abort situation. If the direct trajectory provides for a free M. O. return, then immediately after a capture maneuver at the planet followed by an optional short period of staytime, the planets will be appropriately alined for a near minimum energy M. O. return to Earth. Consequently, an inherent abort capability shortly after capture is provided for a velocity requirement not much greater than is required for a nominal return with minimum energy. An option is available in real time for a short staytime and a long flight time return (a NO-GO or abort decision) or for a commitment to a long staytime to await the next minimum energy opportunity.

The advantages of the M. O. transfers are based on operational considerations to provide options to a nominal mission plan which does not require their use. These advantages may be achieved without an appreciable penalty or constraint on a nominal mission. The advantages do make them worthy of consideration when direct double Hohmann transfers are considered, particularly in the Mars case because the direct flight which provides the Hohmann transfer nearly provides the M. O. Hohmann return (with some variation and velocity penalty).

As the total mission duration is reduced significantly below that required for the direct double Hohmann transfer, the M. O. transfer becomes less attractive; or because of the large minimum M. O. transfer time, it may not be applicable. The M. O. transfers need not be considered when the allowable flight time is less than the values shown in the preceding sections.

6.0 CONCLUSIONS

The data presented in this internal note have shown that the minimum multiple orbit transfer time for Earth-Mars missions is 567 days or 500 days, based on whether the minimum acceptable perihelion distance coincides with the orbit of Venus or the midpoint between the orbits of Mercury and Venus. For Earth-Venus missions, the minimum M. O. time is 412 days or 324 days for the same restrictions. In interplanetary mission studies, an M. O. trajectory need not be sought for flight times less than these, or for values for other constraints given in the included figures. The minimum values of the flight time can be achieved only if the aphelion is approximately 0.05 AU greater than the orbit of the outer planet and can be achieved only once in a synodic cycle of the planet of interest.

Although the flight time for an M. O. transfer is greater than required for a minimum energy transfer, there are some advantages to the M. O. transfer that may be achieved without severe penalties in velocity requirements or compromises in a nominal mission plan. The advantages of the M. O. transfer are that it provides options which permit direct transfers to the planet and maintains a free M. O. return capability and a near minimum energy abort capability shortly after capture at the target planet. Because the M. O. transfer requires more than minimum energy except in a special case, some penalty must be paid for these advantages, but not a large one if the mission is carefully designed.

The M. O. transfer offers no particular advantage in terms of reduction in total mission time because of tradeoffs between staytime and flight time. In addition, there is no gross change in launch opportunity because of similarities with the direct double Hohmann mission. However, more precise studies of these missions would show some shifts of launch dates and total time.

Feasibility of these M. O. missions has been compared primarily with the feasibility of direct double Hohmann missions because of the several similarities; flight time requirements preclude a comparison with the short total time missions which have considerably greater velocity requirements. It appears to this author that the M. O. trajectories are feasible only in an operational sense and require consideration only as alternates to a minimum energy mission. However, to study all possible applications was beyond the intended scope of this study; for example, consideration has not been given, to the very complex possibilities of M. O. trajectories combined with Venus swingbys for a Mars orbital mission. Conceivably, a gravity turn at Venus while on an M. O. trajectory between Earth and Mars creates an entirely new realm of possibilities, with new questions and new answers.

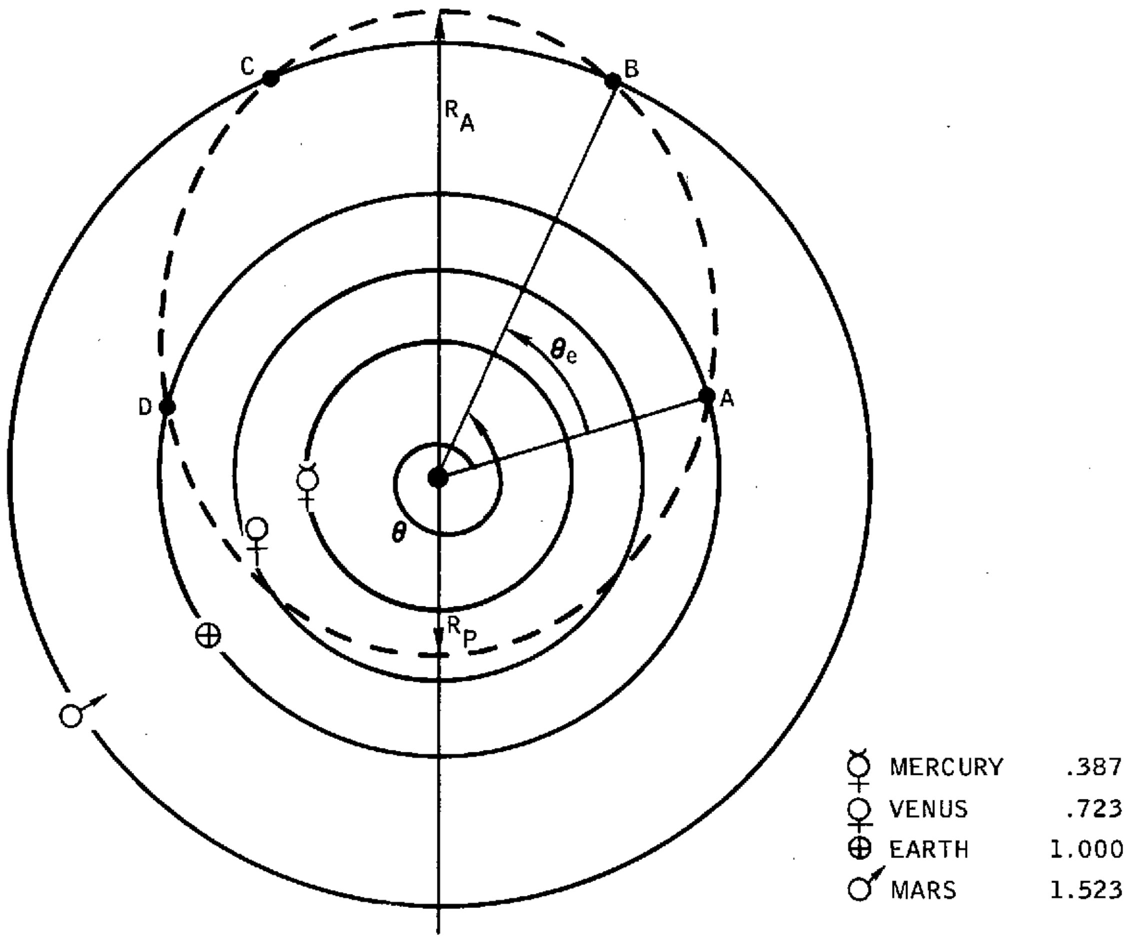
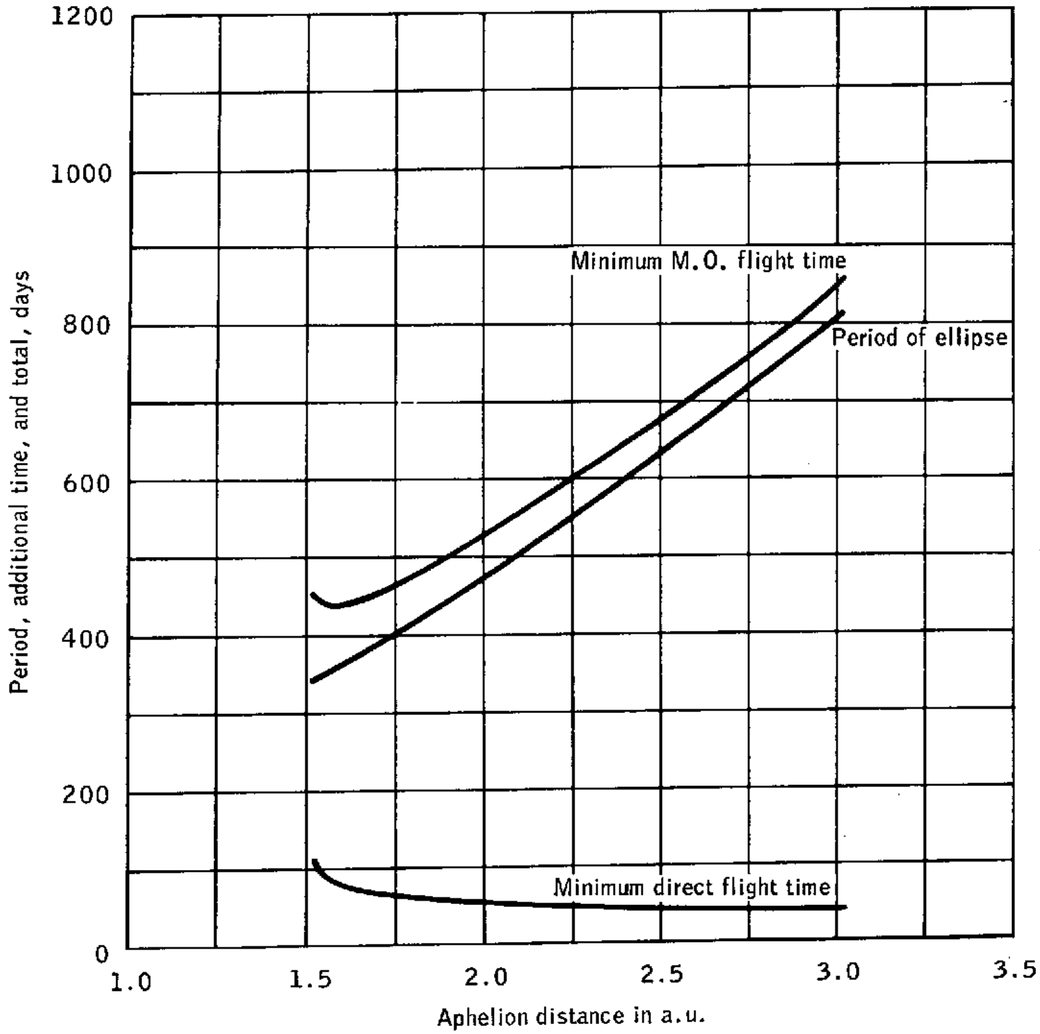
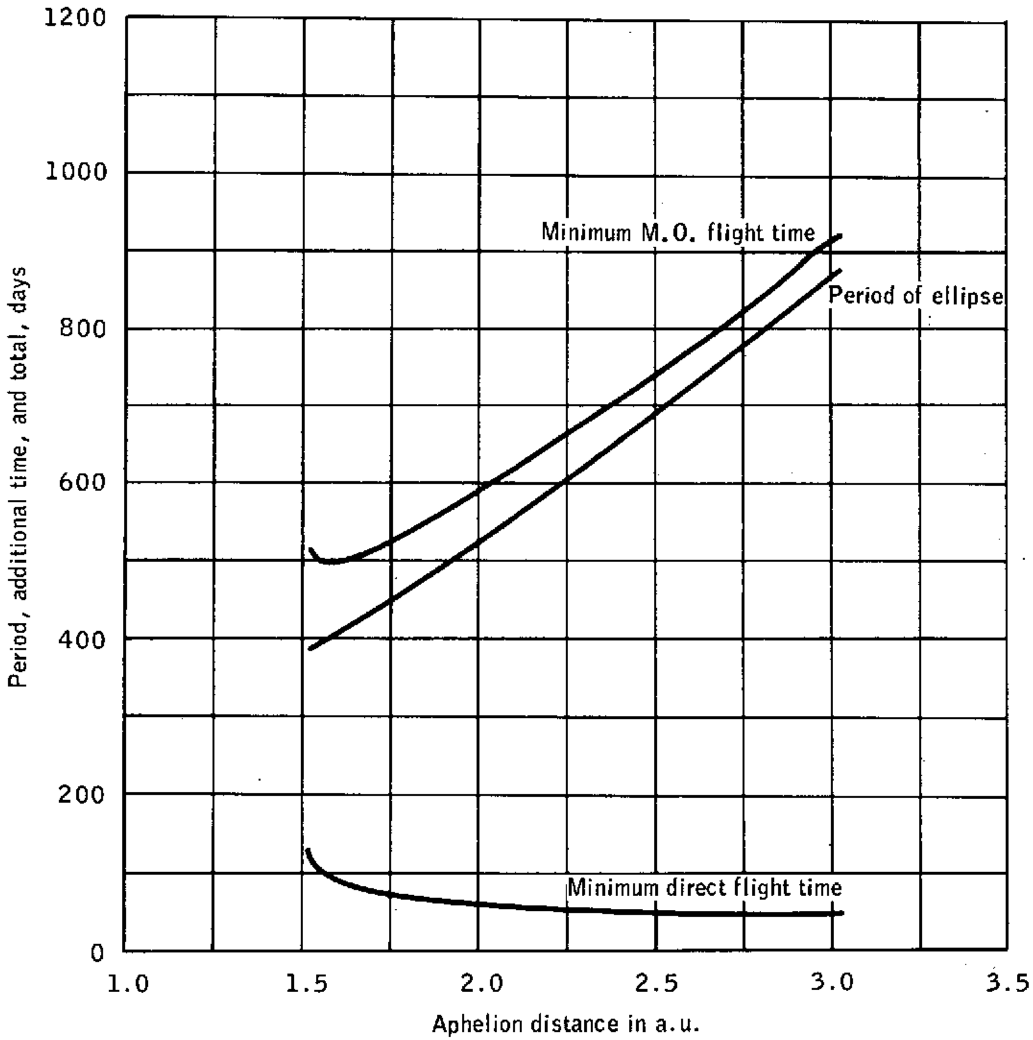


Figure 1.- Geometry for a multiple orbit interplanetary transfer trajectory.



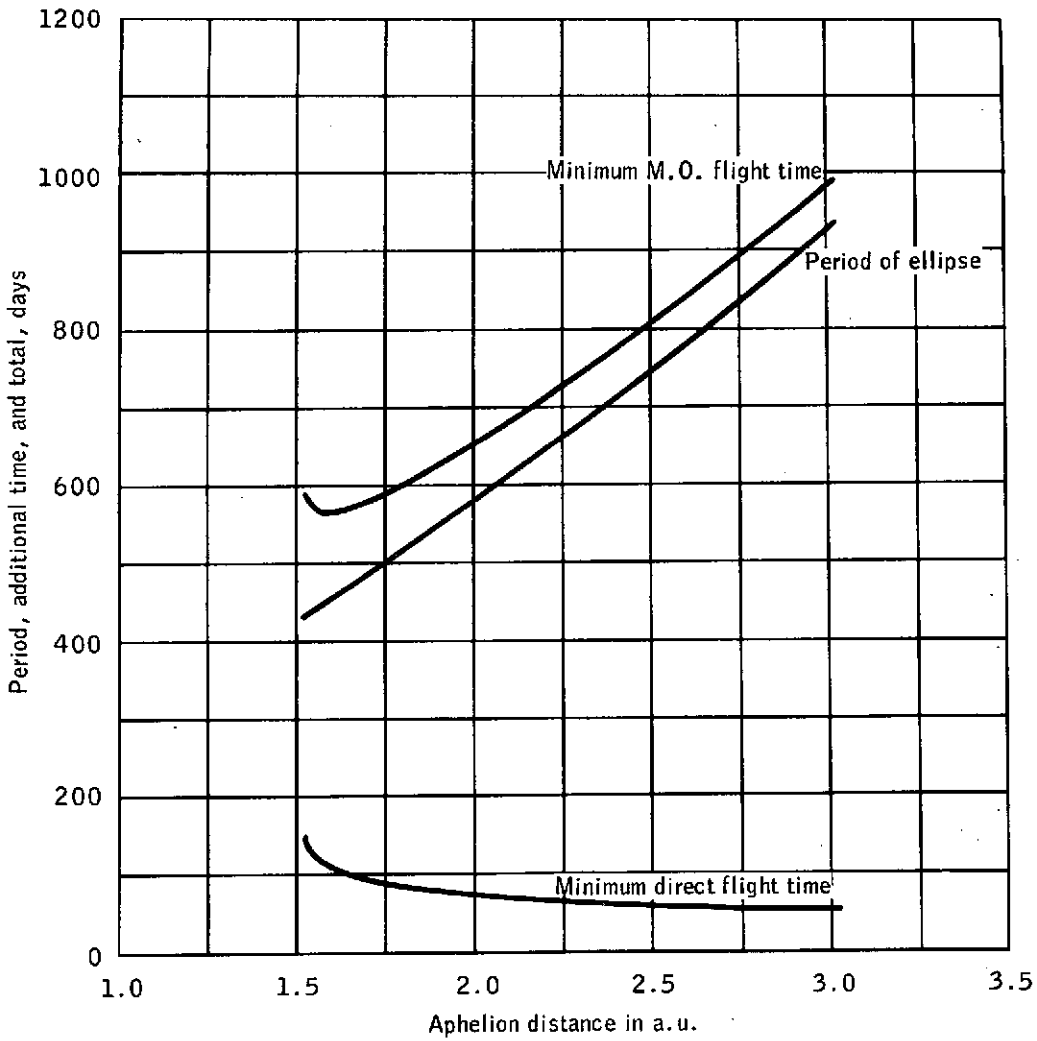
(a) Perihelion is the orbit of Mercury (0.387 a.u.).

Figure 2.- Flight time for a direct transfer, one revolution, and the minimum time multiple orbit transfer between Earth and Mars.



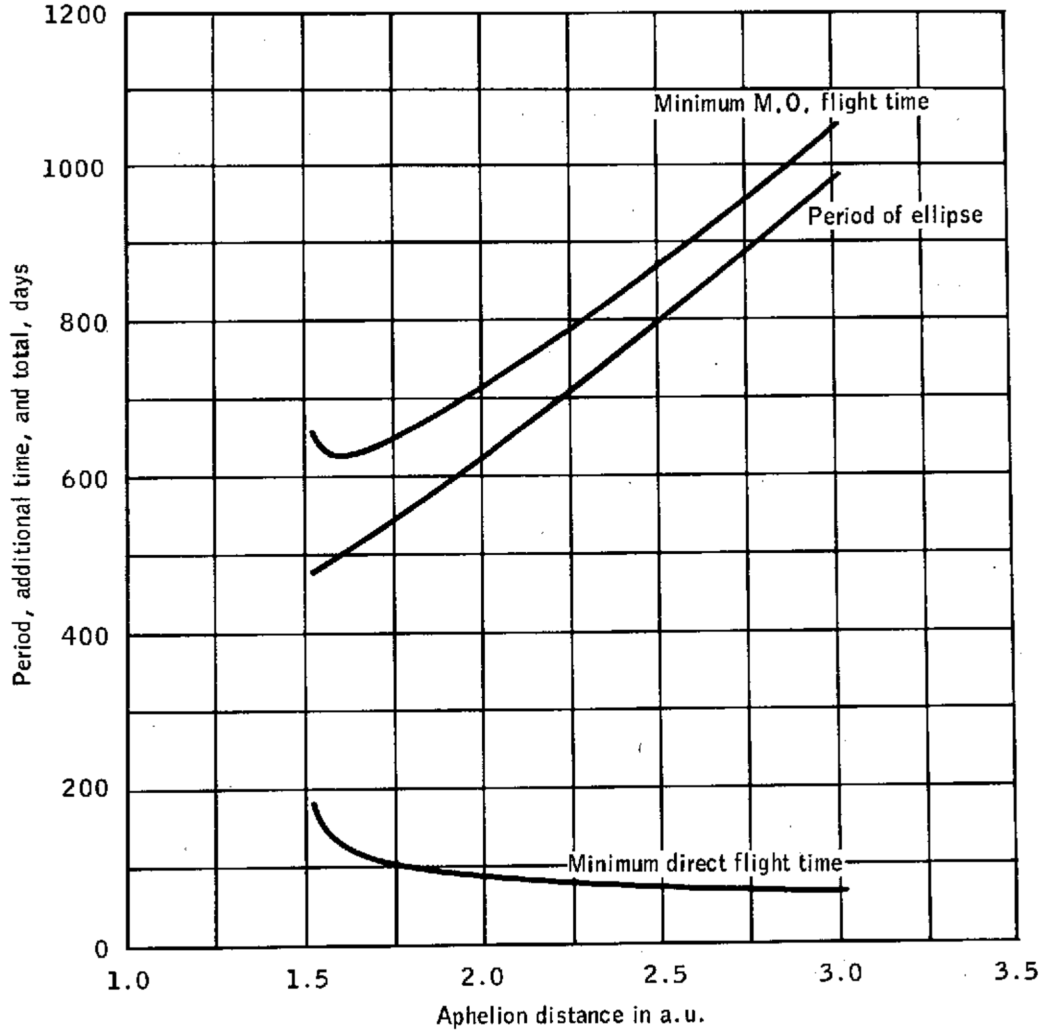
(b) Perihelion is between Mercury and Venus (0.555 a.u.).

Figure 2.- Continued.



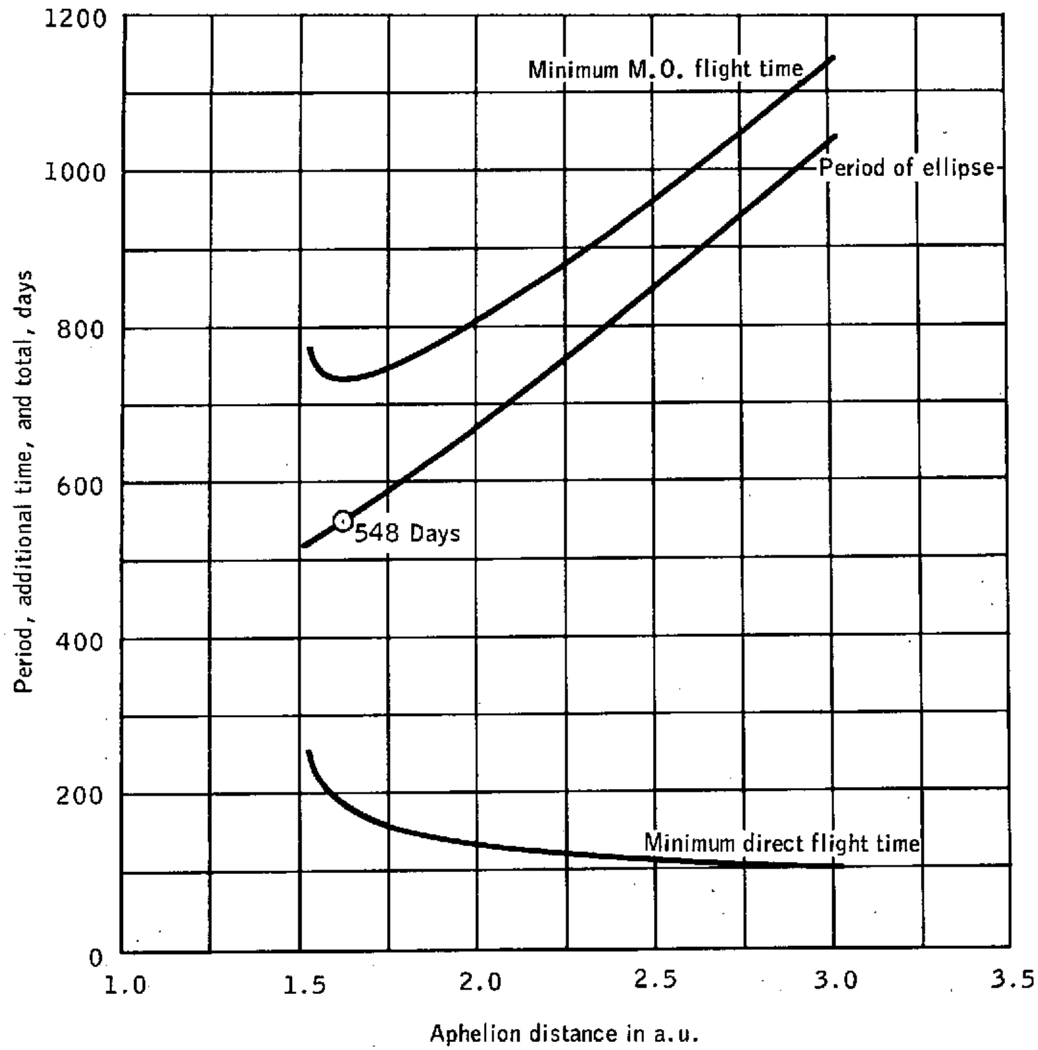
(c) Perihelion is the orbit of Venus (0.723 a.u.).

Figure 2.- Continued.



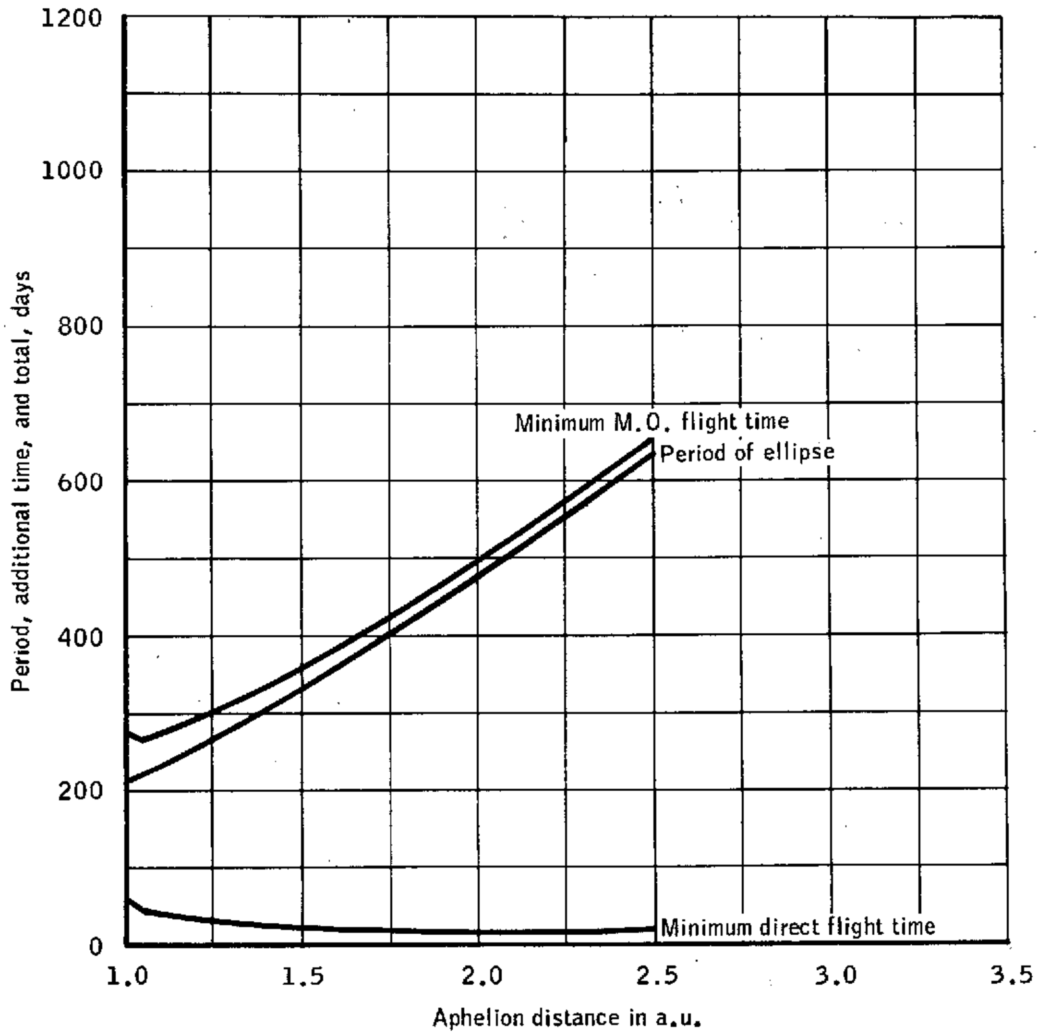
(d) Perihelion is between Venus and Earth (0.861 a.u.).

Figure 2.- Continued.



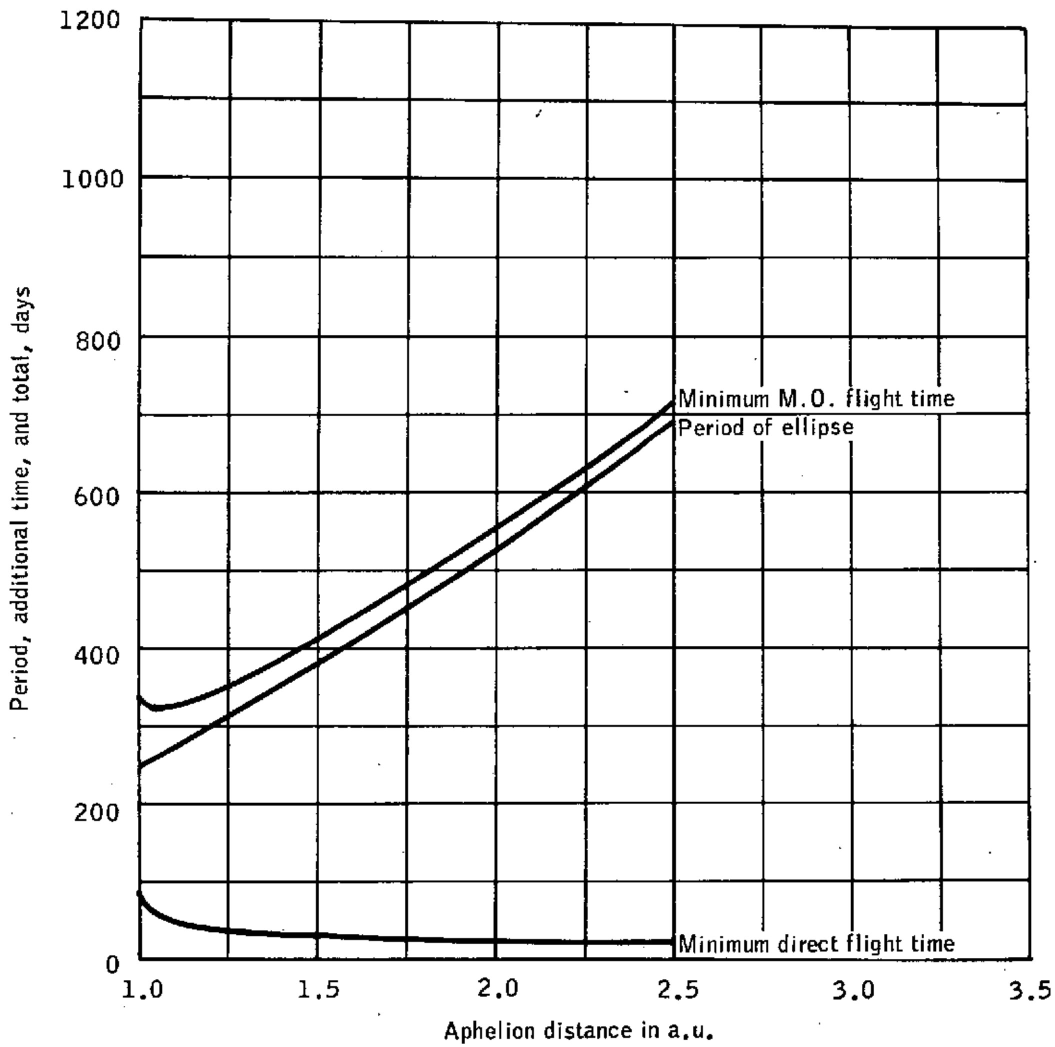
(e) Perihelion is the orbit of Earth (1.000 a.u.).

Figure 2.- Concluded.



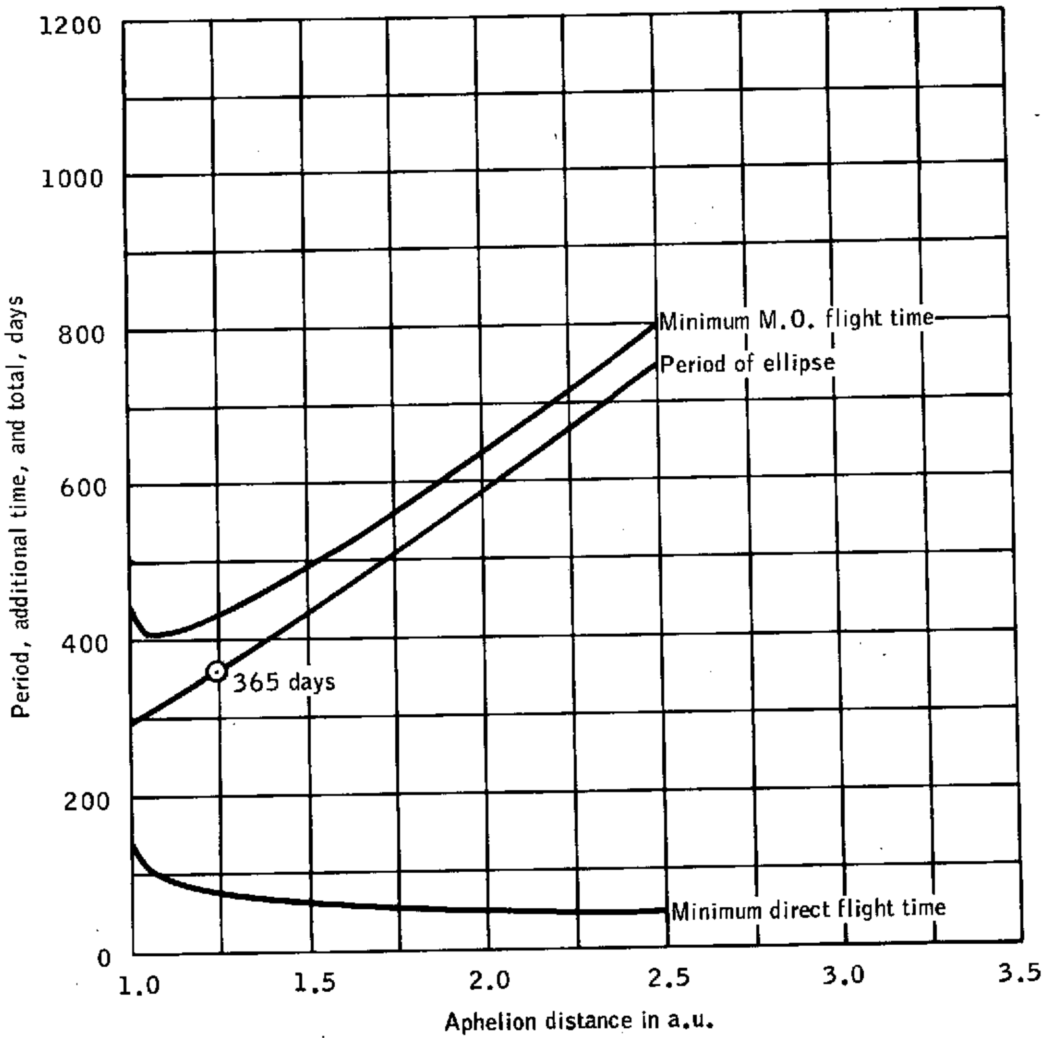
(a) Perihelion is the orbit of Mercury (0.387 a.u.).

Figure 3.- Flight time for a direct transfer, one revolution, and the minimum time multiple orbit transfer between Earth and Venus.



(b) Perihelion is between Mercury and Venus (0.555 a.u.).

Figure 3.- Continued.



(c) Perihelion is the orbit of Venus (0.723 a.u.).

Figure 3.- Concluded.